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THE EC9 APPROACH FOR SHELL BUCKLING

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ABSTRACT

The new Part 1-5 of Eurocode 9, developed within the activity of CEN/TC 250/SC9 Committee chaired by F.M. Mazzolani, is the very first issue specifically devoted to shell structures made of aluminium alloys in the field of European codification. Because of this, it fills up an important gap and marks a significant advance in the codification on shells for both civil and industrial applications. In the same way, it represents the endpoint of the whole path followed by the Eurocode 9, which has now reached the stage of its full development. This paper summarises the buckling issues of the new code while critically pointing out, at the same time, the relationship with the corresponding rules given in EC3 (prEN1993-1-6) for steel shells.

1. INTRODUCTION

The pr1999-1-5 “Shell Structures” represents the latest document produced within the activity of the CEN/TC 250/SC9 Committee (chairman F.M. Mazzolani) and developed after the need for a specific code facing shells had been explicitly expressed by the European Aluminium Industry [1]. The main frame of the prEN1999-1-5 “Shell structures” was originally drafted on the EC3 model of the corresponding ENV1993-1-6 (1996), devoted to shell structures made of steel [2], [3]. This part, like its updated version prEN1993-1-6 “Strength and stability of shell structures”, consisted of a main text and four annexes, dealing with general rules on analysis and design of unstiffened shell structures, including stress and deformation calculation rules (Annexes A, B and C) and specific rules for buckling (Annex D). The prEN1993-1-6 is supplemented by more applicative parts, dealing with silos (prEN1993-4-1) and tanks (prEN1993-4-2), facing specific problems related to these structural categories. Even though in a marginal way only, also documents prEN1993-4-3 (Pipelines), prEN1993-3-1 (Towers and masts) and prEN1993-3-2 (Chimneys) deal with some aspects typical of shells. As a whole, some hundreds of pages are devoted in EC3 to shells, well covering the most relevant aspects of shell design and execution.

When facing the problem of aluminium shells, it was soon recognised that the layout of prEN1993-1-6 was not sufficient to cover all problems related to shell design, it being to be supplemented on one hand, by rules on stiffened shells and, on the other hand, by specific provisions for aluminium alloy shells, such as inelastic buckling, imperfection sensitivity, effect of welding, and so on. As a consequence of this, the new document on aluminium shells was completely reshaped in order to point out the peculiar aspects typical of structures made of such materials, including specific constructional issues. It was therefore decided to create a document in which the main text recalled to some extent the one of prEN1993-1-6, even though with proper changes and amendments, whereas the annex dealing with buckling was largely rewritten and divided into two parts (Annex A and B), in order to include structural configurations such as spheres, toriconical and torispherical shells, as well as stiffened shells. Rules on toriconical and torispherical shells were set out on the basis of the German Code DASt-Richtlinie 017 “Beulsicherheitsnachweise für Schalen” (1992). Annex C was added on buckling relevant geometrical tolerances whereas, because of their general content, Annexes A, B and C present in prEN1993-1-6 were not included in the new prEN1999-1-5 but just referred to.

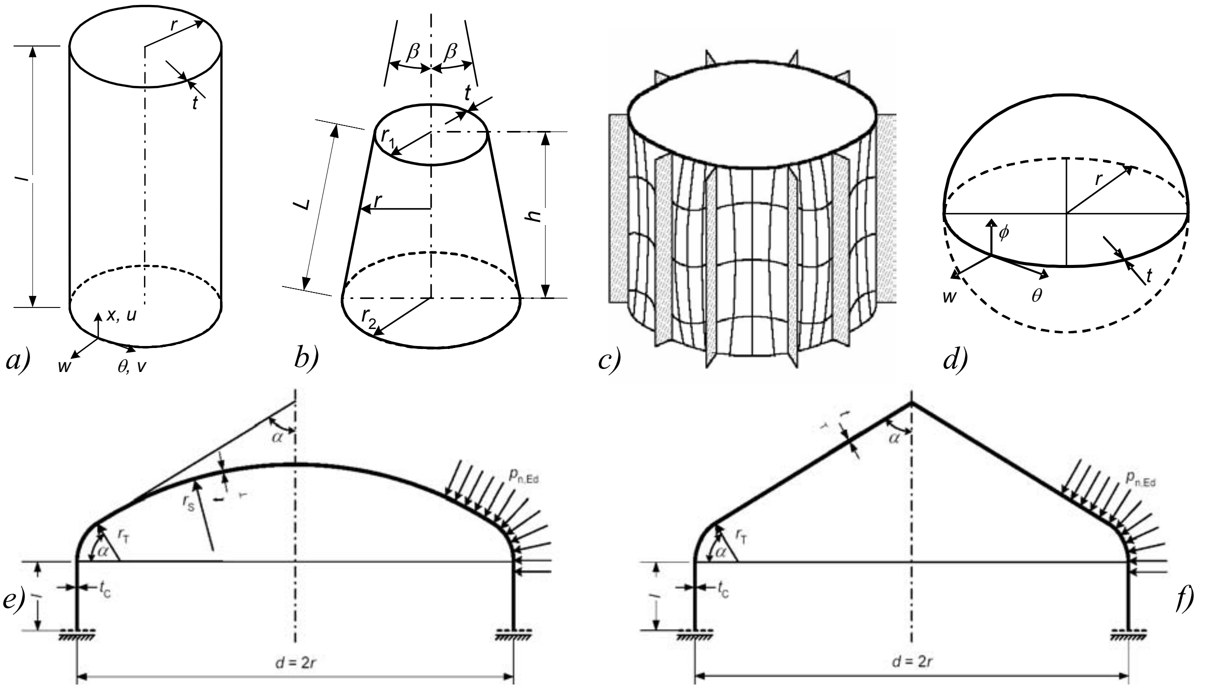

Figure 1: The structural configurations covered in prEN1999-1-5

2. BUCKLING ISSUES

Buckling is the most relevant issue in the codification on shell structures. This may be considered as a consequence, on one hand, of the difficulty to evaluate the very high imperfection sensitivity of such structures, on the other hand, of the problem to take into account the effect of material plasticity. As far as imperfection effect is concerned, both prEN1993-1-6 and prEN1999-1-5 follow the traditional, empirical "Lower Bound Design Philosophy", according to which a knock-down factor of buckling loads, usually denoted by α, is introduced in order to set a lower limit of the scattered experimental data. Similar to prEN1993-1-6, general shell-loading combinations are allowed in prEN1999-1-5 provided fully non linear F.E.M. analysis is used for considering both imperfection and plasticity effect. Nevertheless, specific provisions for buckling are given in prEN1999-1-5 for the following structural configurations (Figure 1):

- Unstiffened cylindrical shells of constant or stepwise wall thickness subjected to meridional (axial) compression, circumferential (hoop) compression and shear (torsion) (Figure 1a);
- Unstiffened conical shells subjected to meridional (axial) compression, circumferential (hoop) compression and shear (torsion) (Figure 1b);
- Stiffened cylindrical shells of constant wall thickness subjected to meridional (axial) compression, circumferential (hoop) compression and shear (torsion) (Figure 1c);
- Unstiffened spherical shells under uniform circumferential compression (Figure 1d);
- Unstiffened toriconical (Figure 1e) and torispherical (Figure 1f) shells under uniform external or internal pressure.

A first attempt to adapt the EC3 approach to aluminium shells was proposed by Mazzolani & Mandara [4], [5], by modifying the parameters provided in the piecewise formulation of buckling factor χ given in prEN1993-1-6. Nevertheless, this led to an unjustified excess of conservativeness and, most of all, to a lack of accuracy in the representation of buckling resistance in the elastic-plastic region. In addition, this approach would involve the commonly accepted difference between strong and weak hardening aluminium alloys to be completely missing [6]. In order to overcome such limits, an alternative formulation for aluminium shell buckling curves has been presented for the same load cases as considered in prEN1993-1-6 [7], [8]. The proposal is based on the format already adopted for buckling of aluminium members in compression and codified into prEN1999-1-1. Imperfection reduction factors, of course, have been properly modified respect to those given in prEN1993-1-6, even though keeping the same shell execution tolerance classes, also defined as "quality classes" or "imperfection classes". According to EC9 basic approach, alloy distinction has been made in pr1999-1-5 in terms of "Material buckling class", namely Class A and Class B for weak and strong hardening alloys, respectively.

In general form, the design buckling resistance is obtained by:

$$\sigma_{Rd} = \alpha_i \rho_{i,w} \chi_{i,perf} \frac{f_0}{\gamma_{M1}} \tag{1}$$

where α_i is the imperfection factor, depending on the load case, shell slenderness and execution tolerance class, $\rho_{i,w}$ is a factor accounting for welding effect (see section 3), f_o and γ_{M1} are the material characteristic limiting strength and a partial safety factor, respectively. $\chi_{i,perf}$ is the buckling factor for a perfect shell, which depends on the relative slenderness $\overline{\lambda}_i$ of the shell and is given by:

$$\chi_{i,\mathrm{perf}} = \frac{1}{\phi_i + \sqrt{\phi_i^2 - \overline{\lambda}_i^2}} \quad \text{with} \quad \phi_i = 0,5\left(1 + \mu_i(\overline{\lambda}_i - \overline{\lambda}_{i,0}) + \overline{\lambda}_i^2\right) \tag{2}$$

In Equation (2) $\overline{\lambda}_{i,0}$ and μ_i are the squash limit relative slenderness and a numerical parameter depending on the alloy buckling class and loading case. Subscript i is to be replaced by x, θ or τ for meridional (axial) compression, circumferential (hoop) compression and shear (torsion), respectively.

The shell slenderness parameter $\overline{\lambda}_i$ for different stress components is defined in a general way as $\overline{\lambda}_x = \sqrt{f_o / \sigma_{x,\mathrm{cr}}}$, $\overline{\lambda}_\theta = \sqrt{f_o / \sigma_{\theta,\mathrm{cr}}}$, $\overline{\lambda}_\tau = \sqrt{f_o / \sqrt{3}\tau_{\mathrm{cr}}}$ and $\overline{\lambda}_x = \sqrt{n_{\mathrm{x,Rk}} / n_{x,\mathrm{cr}}}$ $\overline{\lambda}_\theta = \sqrt{p_{n,\mathrm{Rk}} / p_{n,\mathrm{cr}}}$, for stiffened and/or corrugated shells, respectively. $\sigma_{x,\mathrm{cr}}$, $\sigma_{\theta,\mathrm{cr}}$ and τ_{cr} are the elastic critical buckling stresses for unstiffened shells, whereas $n_{x,\mathrm{cr}}$, $p_{n,\mathrm{cr}}$ and $n_{x,\mathrm{Rk}}$, $p_{n,\mathrm{Rk}}$ are the elastic critical buckling and the characteristic squash stress resultants for stiffened shells, respectively, all of them given in the Annex A of the code.

If more than one stress components are present under the actions under consideration, the following interaction check for the combined membrane stress state should be carried out:

$$\left(\frac{\sigma_{x,Ed}}{\sigma_{x,Rd}}\right)^{k_x}+\left(\frac{\sigma_{\theta,Ed}}{\sigma_{\theta,Rd}}\right)^{k_\theta}-k_i\left(\frac{\sigma_{x,Ed}}{\sigma_{x,Rd}}\right)\left(\frac{\sigma_{\theta,Ed}}{\sigma_{\theta,Rd}}\right)+\left(\frac{\tau_{Ed}}{\tau_{Rd}}\right)^{k_\tau}\leq 1 \qquad (3)$$

where $\sigma_{x,Ed}$, $\sigma_{\theta,Ed}$ and τ_{Ed} are the interaction-relevant groups of the significant values of compressive and shear membrane stresses in the shell. The values of the interaction parameters k_x, k_θ, k_τ and k_i are: $k_x = 1 + (\chi_x)^2$; $k_\theta = 1 + (\chi_\theta)^2$, $k_\tau = 1{,}5 + 0{,}5(\chi_\tau)^2$, $k_i = (\chi_x\chi_\theta)^2$. Special provisions are made in the code when $\sigma_{x,Ed}$ or $\sigma_{\theta,Ed}$ is tensile as well as for axially compressed cylinders with simultaneous internal pressure (leading to circumferential tension). Interaction formula (3) is given in prEN1993-1-6, too, and results from the modification of a similar expression of ENV1993-1-6, in which the exponents k_x, k_θ, k_τ were constant and not related to the corresponding χ values. This change made this formulation adaptable to aluminium shells as well.

Equations (1) and relevant imperfection reduction factors have been checked against a wide imperfection sensitivity analysis of both unstiffened and stiffened shells, carried out by means of ABAQUS F.E.M. simulation [9]. A large number of imperfection distributions, mostly defined on the basis of a preliminary evaluation of shell critical modes, have been analysed, in order to find out the lower bound of buckling loads for a given imperfection class. Structural imperfections have been represented by means of suitable models [7], [8], able to describe imperfection distributions similar to both axisymmetric and asymmetric critical or postcritical modes. In this way the most severe condition for the buckling response has been investigated, so as to determine a lower bound of the ultimate load carrying capacity as a function of the initial imperfection magnitude.

An accurate curve fitting has been made for imperfection magnitudes corresponding to quality classes as defined in prEN1993-1-6. Some results of the curve fitting are shown in Figure 2, together with the corresponding EC3 curves. Such curves are plotted both as they are given in EC3 and in a modified version, initially checked as a possible alternative suitable to aluminium shells and then recognised as inappropriate [4], [5]. Because of the great scattering observed in numerical buckling data, a further semi-probabilistic analysis has been carried out for the evaluation of the lower bound of buckling loads of imperfect cylinders subjected to axial compression. To this purpose numerical data have been treated in stochastic way, in order to extrapolate lower values of ultimate load, corresponding to a given fractile value (e.g. 5%). Such values have been considered as characteristic lower bound for fitting buckling curves. The *Weibull* extreme distribution has been used to this aim, whose cumulative curve is:

$$P(x)=1-e^{-(\gamma x)^{\beta}} \qquad (4)$$

where γ and β are characteristic parameters to be fitted on the basis of available data. Such distribution is well suited to the description of random variables ranging between 0 and 1 and has been already used for the stochastic evaluation of the buckling load of imperfect cylinders [10]. Parameters γ and β have been estimated on the basis of numerical data as a function of both material and radius over thickness (*r/t*) ratio, according to execution tolerance classes defined in prEN1993-1-6. Typical cumulative curves are shown in Figure 3, where the corresponding *Weibull* curves are also plotted, together with the characteristic 5% lower bound. Because of their conservativeness, complying with a commonly shared policy in codification on shells, buckling curves and relevant imperfection reduction factors given for axially loaded cylinders have been also adopted in the code for spherical, toriconical and torispherical shells.

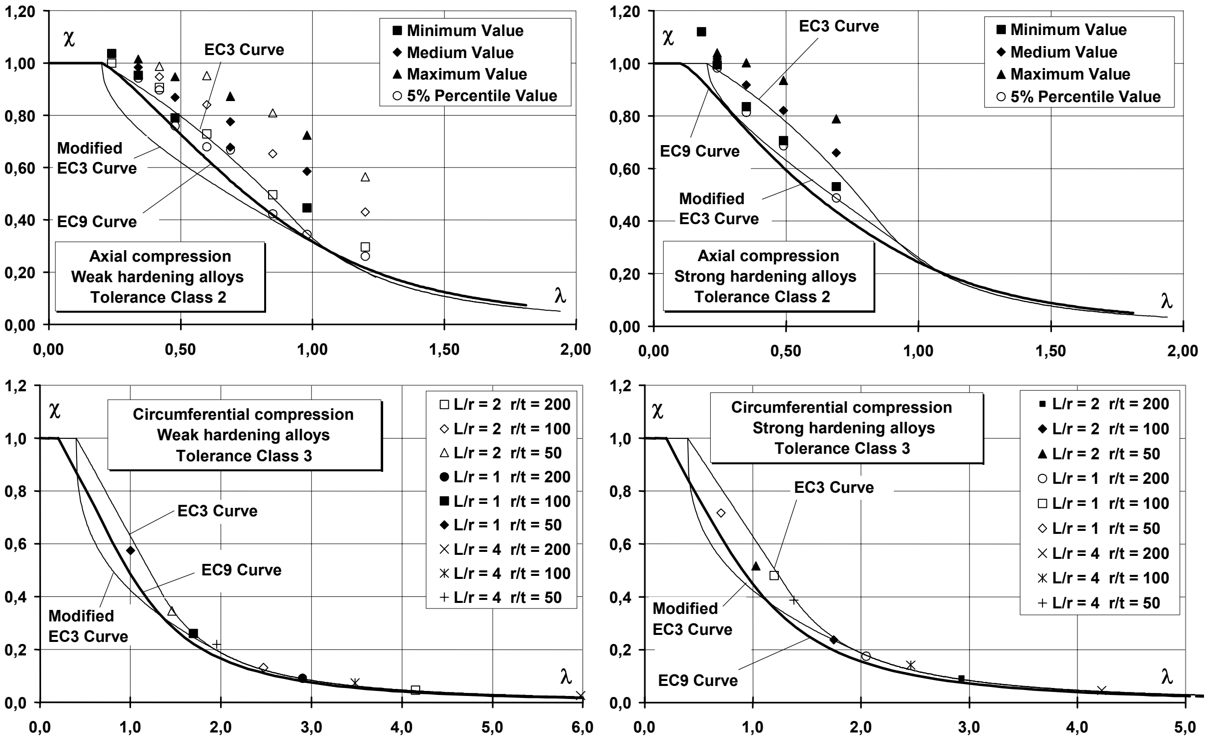

Figure 2: Some EC9 shell buckling curves against the corresponding EC3 curves and numerical simulation buckling data

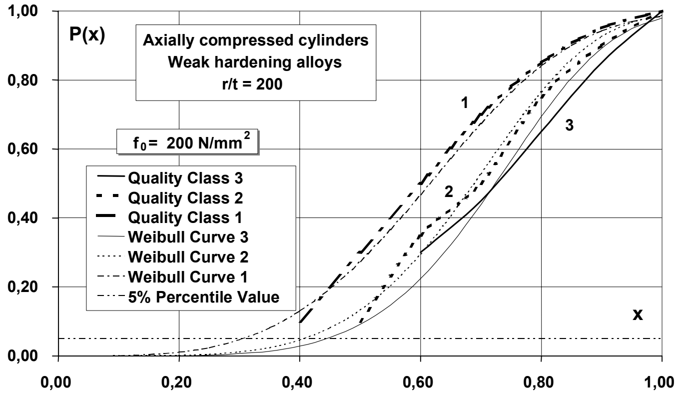

Figure 3: Semi-probabilistic exploitation of numerical simulation buckling data according to *Weibull* extreme law

As an additional provision respect to both ENV1993-1-6 and prEN1993-1-6, a new imperfection class (Class 4), allowed in case of unstiffened cylinders under axial compression only, has been added, in order to consider cylinders buckling in purely plastic range. This is quite liable to occur in case of aluminium alloys, due to the material round-house behaviour, in particular in case of stocky shells made of a relatively low resistance alloy. Compared to elastic buckling, exhibiting the typical "*diamond-shape*" deflection pattern, this kind of buckling is characterised by a purely axisimmetric "*elephant foot*" deflected shape, which involves a lower shell imperfection sensitivity compared to cylinders failing in elastic range. This class, assessed on the basis of an accurate numerical analysis of the shell buckling behaviour [11], corresponds to the nondimensional imperfection limit w_0*/t shown in Figure 4 and is defined in the code on the basis of the dimple tolerance parameter $U_{0,\max}$ given in Table 1.

Also, new expressions for the imperfection reduction factors α_x, α_θ and α_τ have been introduced in prEN1999-1-5, referred to the basic load cases of axial load, circumferential compression and shear (Table 2), respectively. Values of the tolerance parameter Q for axial compression, together with $\alpha_{\theta,\mathrm{ref}}$ and $\alpha_{\tau,\mathrm{ref}}$ for unstiffened cylinders are given in Table 3 as a function of the execution tolerance class of the shell. Based on the results of numerical analysis, a tolerance parameter $Q_{\mathrm{stiff}} = 1.3Q$ has been assumed in case of stiffened or corrugated cylinders under axial load.

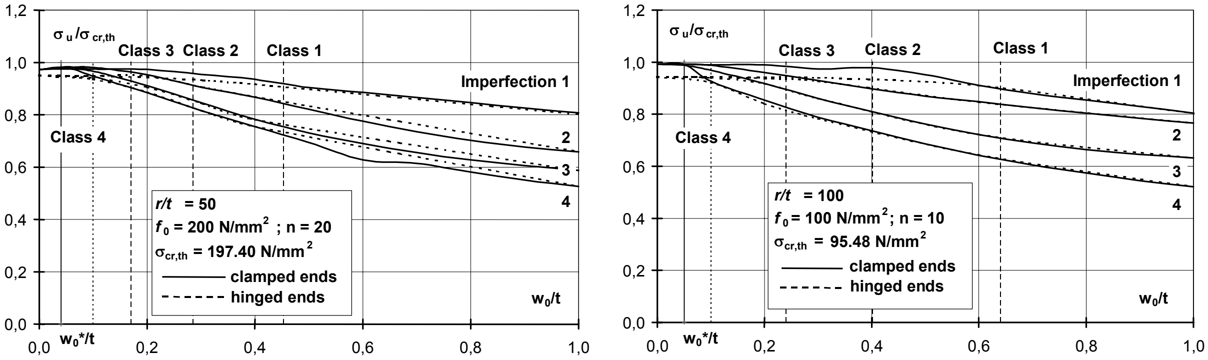

Figure 4: Typical imperfection sensitivity curves for axially compressed cylinders with indication of relevant execution tolerance class

Execution tolerance class	Value of $U_{0,\max}$ for boundary conditions	
	Clamped ends (BC1r, BC2r)	Hinged ends (BC1f, BC2f)
Class 1	0,016	
Class 2	0,010	
Class 3	0,006	
Class 4 (f_o in N/mm^2)	$\frac{1}{f_o}\left(2{,}25\sqrt{\frac{t}{r}}+0{,}01\sqrt{\frac{r}{t}}\right)$	$\frac{1}{f_o}\left(5\sqrt{\frac{t}{r}}+0{,}02\sqrt{\frac{r}{t}}\right)$

Table 1: Values of the dimple tolerance parameter $U_{0,\max}$

Axial compression	Circumferential compression and shear
$a_x = \dfrac{1}{1+2{,}6\left(\dfrac{1}{Q}\sqrt{\dfrac{0{,}6E}{f_{\mathrm{o}}}}(\bar{\lambda}_x-\bar{\lambda}_{x,0})\right)^{1{,}44}}$	$\alpha_{\theta,\tau} = \dfrac{1}{1+0,2(1-\alpha_{\theta,\tau,\mathrm{ref}})(\bar{\lambda}_{\theta,\tau}-\bar{\lambda}_{\theta,\tau,0})/a^2_{\theta,\tau,\mathrm{ref}}}$

Table 2: Expressions of buckling reduction factors α_x, α_θ and α_τ for unstiffened cylinders

Execution tolerance class	Value of Q for boundary conditions		$\alpha_{\theta,\mathrm{ref}}$ and $\alpha_{\tau,\mathrm{ref}}$
	Clamped ends (BC1r, BC2r)	Hinged ends (BC1f, BC2f)	
Class 1	16		0,50
Class 2	25		0,65
Class 3	40		0,75
Class 4	60	50	0,75

Table 3: Values of tolerance parameter Q and imperfection factors $\alpha_{\theta,\mathrm{ref}}$ and $\alpha_{\tau,\mathrm{ref}}$

3. EFFECT OF WELDING ON BUCKLING

A remarkable new issue of prEN1999-1-5 is the allowance for the effect of welding on the buckling load. Even though the reduction mostly affects the 0,2 % proof strength f_o and the ultimate tensile strength f_u of the material, its effects may be not negligible on the compressed parts of a shell susceptible to buckling. As a result, even localised welds placed in areas at risk of buckling may considerably reduce the resistance of the shell due to the presence of a Heat Affected Zone (HAZ) [6]. As shown by refined numerical simulation, the effect of softening due to welding is more significant in shells which buckle in plastic range (Figure 5a), where a premature onset of yielding lines can occur along welding lines. For these reasons, the effect of softening due to welding on the shell buckling resistance should be checked for all welds directly or indirectly subjected to compressive stress. The severity of softening due to welding is expressed through the reduction factors $\rho_{o,haz} = f_{o,haz}/f_o$ and $\rho_{u,haz} = f_{u,haz}/f_u$ given by the ratio of either the characteristic value of the 0,2 % proof strength $f_{o,haz}$ or the ultimate strength $f_{u,haz}$ in the heat affected zone to the corresponding one of the parent material f_o or f_u. The reduction $\rho_{i,w} = \chi_i/\chi_{i,w}$ to allow for HAZ softening is expressed in the code as a function of $\overline{\lambda}_i$ by the expression (Figure 5b):

$$\rho_{i,w} = \omega_0 + \left(1-\omega_0\right)\frac{\overline{\lambda}_i - \overline{\lambda}_{i,0}}{\overline{\lambda}_{i,w} - \overline{\lambda}_{i,0}} \tag{5}$$

where:

$$\omega_0 = \frac{\rho_{u,haz} f_u / \gamma_{M2}}{f_0 / \gamma_{M1}} \tag{6}$$

with γ_{M1} and γ_{M2} material partial safety factors. $\overline{\lambda}_{i,w}$ is the limit value of the relative slenderness parameter beyond which the effect of weld on buckling vanishes, given by:

$$\overline{\lambda}_{i,w} = 1{,}39(1-\rho_{o,haz})(\overline{\lambda}_{i,w,0} - \overline{\lambda}_{i,0}) \tag{7}$$

where $\overline{\lambda}_{i,w,0}$ is the absolute slenderness upper limit for the weld effect, which depends on load case, structural material and quality class of the shell. As for buckling curves, subscript i is to be intended as x, θ or τ for axial compression, circumferential compression and shear, respectively.

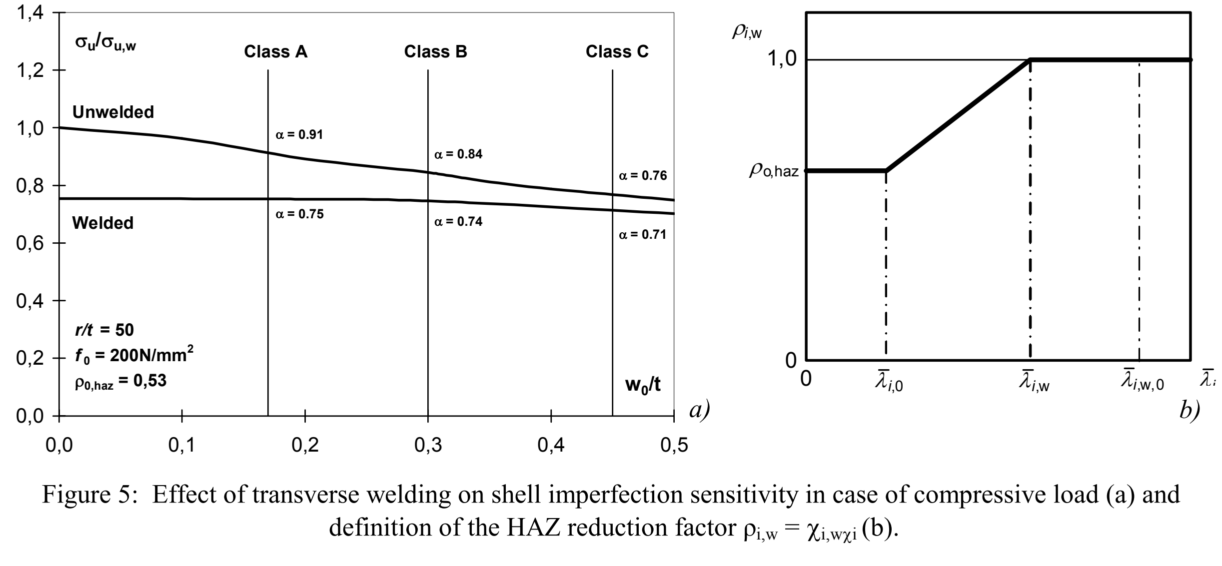

Figure 5: Effect of transverse welding on shell imperfection sensitivity in case of compressive load (a) and definition of the HAZ reduction factor $\rho_{i,w} = \chi_{i,w}\chi_i$ (b).

4. CONCLUSIONS

At the present state of development (Dec. 2007), the shell code can be considered practically at its definitive form. Because of its features, fully harmonised with European major codification, this document does represent a significant landmark in regulation on shell structures. In particular, it is the very first code purposely conceived for aluminium shells, as well as for their application in both civil and industrial field. Shared by most European Countries, it is the result of a long team work made by PT1-1 of CEN/TC 250/SC9 in cooperation with PT1-6, PT4-1 and PT4-2 of CEN/TC 250/SC3, on the basis of a comprehensive scientific background activity for the calibration of buckling curves carried out at both Universities of Naples. The whole of the above activity also earned official acknowledgements from outstanding international Institutions, such as for example the European Aluminium Association (EAA) which, in the name of European Aluminium Industry, expressed great appreciation for the result obtained.

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